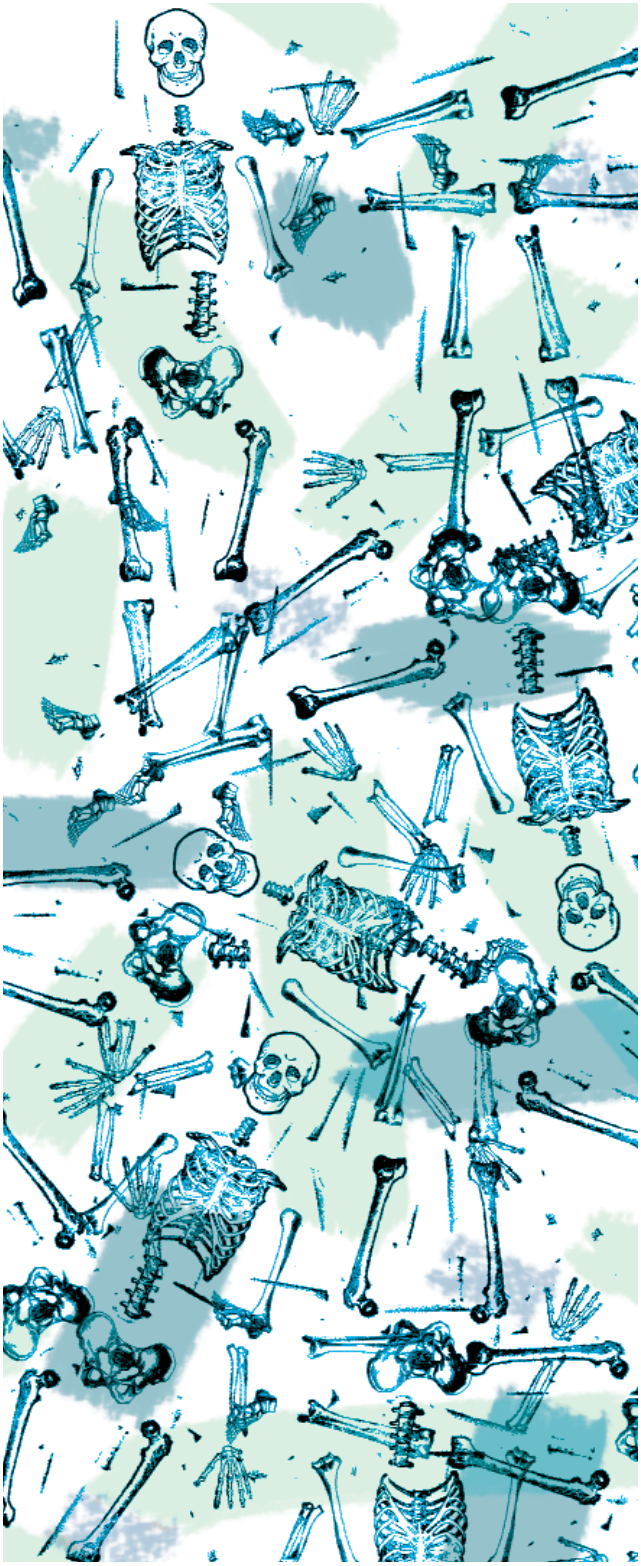
Market Value of Mortality Liabilities



What is the market value of a portfolio of death risks?

This question, posed at the recent Institute seminar on Financial Economics, is answered by the Capital Asset Pricing Model (CAPM) as inclusive of a profit margin of about 5% to 50% on top of expected claims cost. This is at odds with assertions that CAPM implies a nil profit margin.

CAPM Model

For present purposes assume that the CAPM model is valid. Whether this assumption is true is a question to be debated at another time.

For an asset portfolio P that is a subset of the market portfolio M, let

R_P = expected portfolio return

R_F = risk free rate

σ_P = standard deviation of the portfolio return

$\theta = (R_P - R_F) / \sigma_P$

Differentiating θ with respect to each security in the portfolio and setting the derivative to zero gives a set of simultaneous equations (with one equation for each security in the market) that when solved give the CAPM model for the risk margin applicable to the individual securities.

The CAPM derivation applies only to a security included in the market set. Hence mortality risk must be included in the market set before the risk margin is derived. If mortality is not included in the market set then the common error of asserting that a nil correlation with the share market implies a nil risk margin follows. This erroneous conclusion arises because of an implicit attempt, in breach of the CAPM assumptions, to differentiate θ with respect to a security outside the market set (see Modern Portfolio Theory and Investment Analysis, Elton & Gruber, John Wiley)

To include mortality risk in the market set, we proceed as follows. The figures are indicative only, to show that mortality has a price.

ASX Stocks

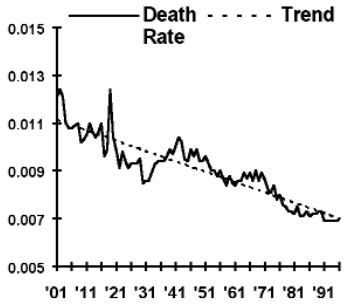
Stocks listed on the ASX have a market capitalisation of about $700 billion. The standard deviation of the ASX All Ordinaries return, i.e. the systemic risk, is about 14% or $98 billion.

Australian Lives

There are about 20 million Australian lives. Assuming a value of $600,000 per life values the population at $12,000 billion.

To estimate the systemic mortality standard deviation, the mortality rates of the whole Australian population for the period 1901 to 1996 have been graphed below.

Mortality rate fluctuations around the trend line are normally distributed with an unbiased estimate of the standard deviation s=0.00057. The standard deviation of the value of the population is then $6.8 billion.



ASX Stocks & Australian Lives

The combined value of ASX stocks and Australian lives is $12,700 billion. ASX stock returns and Australian lives mortality are uncorrelated, so that the combined standard deviation is $98 billion. Covariance of ASX stocks with the total is 9,604 and for Australian lives covariance is 47. Correlations are 0.998 and 0.070 respectively.

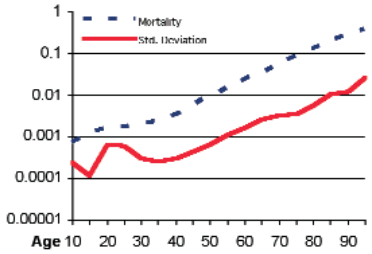
If the ASX CAPM risk premium is 6.50% then the risk premium for the total is 6.532%. The risk premium for Australian lives is then 0.032%. This risk premium level gives a probability of adequacy of Pr{Z<0.00032/0.000572} = Pr{Z<0.56} = 0.65, i.e. 65%. The conceptual basis is the one used by APRA for general insurance, but the probability of premium adequacy is 65% instead of the 75% required for reserves.

If an Australian life had a value of $750,000 then the risk premium increases from 0.032% to 0.040% and the probability of adequacy becomes 75%. The APRA level is reasonable.

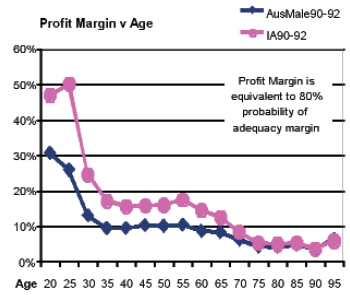
The population average mortality rate is 0.6%. The CAPM risk premium is equivalent to a profit margin of 0.032/0.6=5.3% of the expected mortality cost.

Profit Margin by Age

Systemic variation in mortality rates by age is graphed below on a logarithmic scale.



The variation by age can be used to calculate profit margins. The profit margins according to age are shown (following graph) for group life mortality using Australian Male Lives 1990-92 and for insured lives using Australian Insured Lives 1990-92.



Reserves

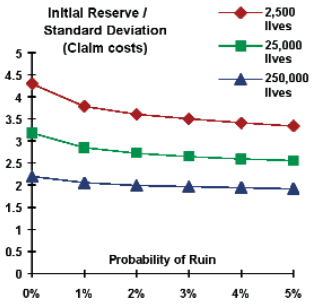
The discount rate is a risk-free rate in a single factor CAPM model. In a multi-factor model, the illiquidity of the liabilities warrants an increase in the discount rate of about 2%.

Mortality variance risk is carried by both the price margin received and by the initial reserve level.

The level of diversification must also be allowed for.

For a particular claim size distribution, the following graph plots a range of probabilities of reserve adequacy against the reserve level (measured as a fraction of the standard deviation of claims) for a one-year case and for a 12.5% margin.

In practice, the standard deviation of aggregate claims cost is a fraction of the average aggregate claims cost, so that the level of required reserves differs little with the probability of adequacy. Hence a suitable reserve level can be closely approximated.



CONCLUSIONS

Premiums

A CAPM model can be used to derive a profit margin required as the price of mortality fluctuation risk, as a profit margin of on top of expected claims cost.

The margin or price received for carrying mortality fluctuation risk is determined by the systemic part of the mortality fluctuation risk. The profit margin in premiums varies with age.

Reserves

The market value of a portfolio of mortality risks is the sum of the average or expected claims payable, plus the price of the systemic part of the risk inherent in the claims variability, all discounted to allow for the time value of money.

The reserve level required does not differ much over the 95% to 99% probability of adequacy level. ▲

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