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Risk Discount Rates for Market Valuation of Life Insurance Business

by

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1 Introduction

For general insurance, PS300 paragraph 13 reads "A risk free rate of return refers to the expected rate of return on a matched portfolio of investments with minimal risk and paragraph 35 reads "The risk free rate of return should normally be the starting point for determining the appropriate risk discount rate". This gives a risk discount rate for liabilities independent of the yield on the actual asset portfolio; rather it depends on the expected rate of return on a matched portfolio of investments with minimal risk. The nature of the matching asset portfolio is dependent on the nature of the risk inherent in the liabilities. Consistent with this, the finance literature also attributes to liabilities a risk discount rate determined by the risk inherent in the liabilities.

In contrast Margin on Services (MOS) actuarial practice attributes to liabilities a discount rate varying with the yield on assets and independent of the risk inherent in the liabilities.

This note is directed at determination of risk discount rates for calculation of market values as distinct from MOS discount rates based on the yield on the actual assets held.

Using the PS300 approach this paper gives the conceptual framework, implementation methodology and illustrative results for risk discount rates appropriate to life insurance products in Australian conditions.

The yield on a theoretical asset portfolio matching the liabilities may be determined using a statistical or econometric model. Models used to estimate discount rates generally have their origins in single factor or multiple factor statistical regression models. These models may then have an economic theory developed to explain (approximately) the observed results. The common choices, the single factor Capital Asset Pricing Model and the multiple factor Barr Rosenberg type models are discussed in Appendix F.

As an ancillary result, the conceptual framework for determination of risk discount rates for valuation of life insurance liabilities also provides the basis for determination of required profit margins and fluctuation reserve levels.

2 Application to Life Office Products

2.1 Generalised Product Description

The margins emerging from life insurance business are in respect of:

- new business sales risk,
- deferred acquisition cost (DAC) recovery,
- reinsurance margin against demographic experience fluctuation,
- a risk margin on the fluctuation buffer capital assets as compensation for the investment fluctuation risk borne,
- expense margin to provide for the economic rent on the administrative and working capital assets.

Distributable profits will be negative at inception as capital is provided for DAC and fluctuation buffer reserves. Distributable profits will be positive in later years as this capital is recovered with interest. The discount rate at which the distributable profits are valued, incorporating both financial risk and asset maintenance costs, is a weighted average of the yield on the new business sales risk, DAC, the margin to compensate for demographic fluctuation risk, the yield on the fluctuation buffer reserves being the investment return geared up by the liability fluctuation risk, together with the return on administrative capital. This average will change over time for any given tranche of new business, because DAC will decrease whilst other components, e.g. administrative return, remain constant.

For conceptual simplicity each type of risk may be given a profit centre and a valuation discount rate developed for that profit centre. If different products combine risks from distinct profit centres in different proportions then the derivation of valuation discount rates from a combination of the criteria of the distinct profit centres gives a consistent approach between products.

2.2 Risk Discount Rate for New Business Sales Operations

How should the financial risk and asset maintenance cost associated with making sales be allowed for? To determine a financial risk discount rate and an asset maintenance cost (equal to the ruin contingency margin) for sales it is conceptually convenient to consider a sales profit centre which is allocated an initial amount of capital. Each year capital is expended on marketing. Sales made generate a margin (in the future annual expense allowance) to recoup the initial financing strain (DAC). The DAC asset is "sold" by the sales profit centre to the life office. The financial risk margin and ruin contingency margin values equal the excess of the DAC "sale value over the marketing expenditure generating it.

As an example consider Australian investment account business over the last 10 years. A yield about 5% less than the bond rate is appropriate for financial risk. For sales profit centre capital of one year's DAC sold and for a ruin probability of 2.5%, a margin of 20% of DAC generated is required. This is a commercially acceptable position. Details of the calculation are given in Appendix B. (Note that since this paper is focussed on risk discount rates the derivation of sales centre capital of one year's DAC sold is not set out herein.)

If the margins in the new business are sufficient only to meet the above criteria then the consequence is that new business generates no value additional to the capital provided.

At the time a tranche of new business is sold its value equals the value of premiums payable plus the value of the capital provided for DAC, fluctuation reserves etc, less the value of future policy holder liabilities and administrative and asset maintenance expenses, with each component valued at its appropriate discount rate.

2.3 Risk Discount Rate for Future New Business

What risk discount rate should be used for discounting back from the projected future date of sale to the valuation date?

If the availability of present terms is guaranteed for future sales then a valuable option is provided in that a policy holder can buy on the better of present and alternative future market rates. In that case the value of the option should be included as a reduction in the value of new business.

The option is particularly significant if it has been assumed that the life office will earn extraordinary profits in excess of the required return on capital. Due to increased competition attracted by the excess returns over the cost of capital, when future policy holders exercise their option to purchase at then current market rates, the excess return and its capitalised value may disappear. In the event that extraordinary profit is presently being generated, given research indications that such profits disappear over about five years, (see e.g. Poterba & Summers) capitalisation of current experience for a longer period is imprudent.

Of equal importance is the conclusion that, if the terms on which future sales are made are market rates lacking any extraordinary profit element, then no risk is incurred during the deferment period and so only the time value of money needs to be allowed for. A bond rate is then appropriate. This rate is likely to be appropriate when product establishment costs are being recouped over future tranches of sales.

Current valuation practice adds to the embedded value of existing business a value of new business. Allowing for the increased yield required to compensate for the sales risk to which the capital involved is subjected, there is generally no excess yield to be capitalised as a value to new business.

2.4 Risk Discount Rate for Surrender Risk & Mortality Risk

Deferred acquisition costs (DAC) are recovered through a margin which varies by product line. DAC may be subject to risk of loss if surrender and death rates are higher than originally budgeted for: this depends on product design. For example, if on early surrender the unrecouped DAC is retained by charging a suitable early surrender penalty then DAC recovery is certain; if no surrender option exists then this risk is avoided.

The yield used to value the DAC recovery margin must be consistent with the undiversifiable risk level in both the surrender rate arid the bonus crediting rate. DAC values are thus correlated with market yields and so the yield may be expected to exceed the bond rate.

To allow for surrender risk the risk discount rate for the DAC recovery margin should be increased by about 0.5% gross or 0.3% net of tax. Details are given in Appendix C.

A reduction in the discount rate for liabilities of about 0.2% suffices to provide for the mortality risk applicable to the DAC asset. A corresponding 4% increase in the DAC discount rate should be made to allow for mortality risk. Details are in Appendix D.

2.5 Cost of Capital Guarantees

As an example, suppose that a property is purchased for $100,000 and rented at a market yield of 12%, at a time when debt finance is available at 6%. The market value of the property is equal to the capitalised value of the future rental income of $12,000 per annum, i.e. $12,000 / 0.12 = $100,000. That market value exists independently of whether ownership is financed by equity or debt. If the owner of the property financially gears the property by borrowing $50,000 of debt at 6%, then net rental income is $9,000 per annum after deduction of $3,000 per annum interest cost. This income needs to be capitalised at a yield of 18% to give a value of $9,000 / 0.18 = $50,000. This must be so, otherwise the sum of the values of the debt and equity would exceed $100,000. This would contradict the proposition that a single market value exists. Gearing has increased the level of risk applying to the residual equity and so the risk adjusted discount rate has increased from 12% to 18%.

Consider the example of a flexible insurance policy credited with annual bonus from investment earnings and with a surrender value payable at call. If the liability is matched by investment in interest bearing assets redeemable at face value at call then no asset liability mismatch exists and no mismatch (i.e. resilience) reserves are required. If shareholders provide reserves then assets and liabilities may be mismatched by investment for example in shares. The shareholders bear the risk attached to capital value and income volatility in respect of both their own assets and policy holder assets. The formula for calculation of geared returns shows that the shareholders required rate of return is the return on their share invested reserves plus the excess of share returns over the return on an asset portfolio matching the policy holder liability. This geared up return must be attributed to the shareholder reserves for those reserves to earn the rate of return required to justify holding those assets at market value without discount.

For capital guaranteed investment account business the investment return in excess of the risk free rate on the policy holder reserves is required as compensation to the fluctuation reserves for increased financial risk and therefore cannot be capitalised as an increase in value. If this excess return is included in distributable profits then there must be a corresponding increase in the financial risk discount rate to leave the fluctuation reserve value unchanged.

Asset maintenance cost, which in this case is the contingency margin against ruin, also needs to be considered. Details are given in Appendix E.

Consider non-participating guaranteed rate investment account business. For reserves of $50 per $100 account balance, at a 2.5% probability of ruin, if the mean return is 2% per quarter then only 1% per quarter may be credited. The cost of protection against ruin is I % per quarter. It is no surprise that little business is written on such terms.

The above analysis can be extended to participating investment account business. The mean contingency margin required to protect against ruin is about 0.07% for reserves of 50% of account balances, which for that reserve level is negligible. The margin may fluctuate over time since the mean is not stable.

For the Commonwealth All Bonds (non-rebateable) Index, Circular 273 reserves were 8.5% of assets. If such a reserve level was held then to satisfy a 2.5% probability of ruin criterion the margin against ruin needed to be 1.0% or more. This margin level is not negligible.

The significant conclusion for participating investment account business is that the business, is, prima facie, able to be credited only with a return at most equal to the risk discount rate appropriate to the risk inherent in that business.

2.6 Administrative Expenses

Capital is invested in business equipment, EDP systems, working capital, etc. An annual fee is received for the provision of administrative services. If costs are less than fee income then a return is earned on the capital supporting the administrative service. If costs exceed fee income then a loss in made. The return on administrative capital should be consistent with the risk involved. A gearing adjustment to allow for lease finance should be made.

Since the expense margin (life office income) is usually a fraction of policy liabilities and is subject to the same volatility, the margin should be capitalised at the risk discount rate used to value DAC. The expense outgo should be capitalised at the same rate if perfectly matched to expense margin income, otherwise it must be decreased to allow for the increased risk arising from fluctuations in experience in respect of the expense income and outgo mismatch, No generalisation appears possible.

The value of an asset is equal to the capitalised value, at the risk discount rate appropriate to the individual asset, of future income (i.e. economic rent) receivable. Net worth assets may comprise independent subsidiary businesses or operating assets used in the conduct of the life office business. Hence the necessity of internal transfer pricing at the economic rent for the use of operating assets. In respect of operating assets the expenses provided for in the liabilities must include the full economic rent of the operating assets. if operating assets are included at market value and credited with an income stream equal to the economic rent then the value of the net worth of the life office becomes independent of whether the operating assets are owned or leased.

2.7 Asset Maintenance Costs

The risk discount rate allows only for financial risk. Moreover, the return receivable for carrying financial risk is only in respect of undiversifiable financial risk. The cost of other types of risk should be added to the liability cash flow as an asset maintenance cost. This separation reduces estimation error in the risk discount rate for financial risk. Separate specific allowance for asset maintenance costs provides conceptual clarity and also reduces estimation error. Treatment of asset maintenance costs as a charge on assets reflects what happens in practice e.g. provision for reinsurance costs for mortality risk.

For example, consider the risk discount rate appropriate to one of the recently privatised electricity distribution companies operating in rural Victoria. The asset is a network of power poles. On average 4% are burnt in bush each year. The insurance cost is 5%, the 1% excess over 4% being the insurer's price for carrying the fluctuations in experience. Suppose that the fire risk is insured, that profit is $10 million p.a. and that 10% is an appropriate rate at which to capitalise future profit so that market value is $100 million. If the fire risk is self insured then profit increases by the insurance profit margin of 1% of $100 million assets, i.e. $1 million. If provision is not made for the notional cost of insurance as an asset maintenance cost then the capitalised value of the business will increase from $100 million to $110 million, a clearly unreasonable result. The value would be unchanged if the discount rate was increased by 1% to 11%. However this is not justified on financial risk grounds, rather the conceptual basis is entirely different and the increase in the discount rate is merely a technique for combining allowance for both the financial risk and the asset maintenance cost.

If asset maintenance cost is not treated as a deduction from asset proceeds then, since asset maintenance costs are not correlated with any financial variable, there emerges an uncorrelated positive residual return. This causes estimation problems with risk discount models which invariably are derived on the assumption that the residual return should average zero.

How may asset maintenance costs be determined? De Moivre showed in the 18th century that an insurance company must charge a contingency margin, in addition to the average cost of the risk, if ruin is to be avoided. If a profit or contingency margin is not provided for then random fluctuations will eventually result in the capital of the life office being lost. The asset maintenance cost provides that margin and may be calculated using collective risk theory as the margin against ruin. An example is given in Appendix AS. The margin is an asset maintenance cost.

The margin is calculated using collective risk theory, originally developed by the Swedish actuary Filip Lundberg between 1909 and 1934. An excellent synthesis of disparate aspects of the theory is given by Buhlmann. That stochastic insurance risk model gives improved results when compared with a simplistic model where random variables are replaced by their mean values. In particular it allows a price to be put on insurable fluctuation risk e.g. the price of fluctuations from bush fire effects in the example above.

3 Individual Products

Each component of the liabilities should be valued at its applicable risk discount rate. Given modem projection techniques, this is in practice not difficult to do, The required rates of return for the various risks provide both product profitability criteria and appropriate valuation discount rates.

Simplified examples below illustrate the use of the above results. Risk discount rates applicable to income allocated to shareholders are derived using a bond rate of 8%. Assets are assumed to be at call so that no gearing risk on solvency reserves need be priced.

3.1 Funeral Bonds (No surrenders)

For a $100 bond with a 12 year expected life:

Marketing expenditure	$4.00	
Marketing profit	$1.00	
Yield	8% – 5% + 25% = 28%	(1)
Deferred acquisition cost financed	$5.05	
Annual recoup including interest	$0.84	
Yield	8% + 4% = 12%	(2)
Average yield before tax	16.4%	

(1) 8% bond yield, less 5% for counter-cyclical financial risk, plus 25% for marketing risk (see Appendix B).

(2) 8% bond yield, plus 4% for mortality risk (see Appendix D).

The deferred acquisition cost in existing funeral bond business will be valued at 12% because the marketing risk has passed. The average yield required on new business is 16.4% (see Appendix G).

3.2 Flexible Insurance Bonds

For a $100 bond with a 6 year expected life:

Marketing expenditure	$4.00
Marketing profit	$1.00
Yield	8% – 5% + 25% = 28%
Deferred acquisition cost financed	$5.05
Annual recoup including interest	$1.22
Yield	8% + 0.5% + 4% = 12.5%
Average yield before tax	18.9%

Details are given in Appendix C.

3.3 Superannuation/Deferred Annuity Bonds

For a $100 bond with a 3 year expected life:

Marketing expenditure	$3.20
Marketing profit	$0.80
Yield	8% – 5% + 25% = 28%
Deferred acquisition cost financed	$4.04
Annual recoup including interest	$1.70
Yield	8% + 0.5% + 4% = 12.5%
Average yield	23.5%

Details are given in Appendix G.

3.4 Solvency Reserves

Capital guaranteed funds provide policy holders with a risk-free investment so that the required and valuation gross interest rate is the risk-free rate of say 8% ignoring expenses. If solvency reserves of 1% of assets are held then the average duration is about 3/4 year and the return is say 8.2%. The geared return required on the solvency reserves is then 28%.

During the 1980's a common hurdle yield was of 15% after tax, i.e. 17.5% to 25% before tax for superannuation or flexible insurance business. These appear consistent with flexible insurance and superannuation gross yields of 11% to 16% above the bond rate, as derived above for new business.

At present the yield on both existing and new business, which for the life insurance industry is predominantly superannuation business, will be around 22% gross or 14% to 18% after tax depending on the mix of product types and the relativity between existing and new business.

If no new business is sold then the yield drops to about 12.5% gross. Solvency reserves should earn about 25% to 30% because of the highly geared risk. Since in a mutual organisation the return above that retained to fund expansion of the business is distributed to members the issue is of no importance in market valuation although the issue is significant for continuing capital adequacy.

For existing business fluctuation reserves are held against the residual fluctuations remaining when using a matched asset portfolio. If such reserves are included in net life office assets then the corresponding asset maintenance cost, i.e. the profit margin provided to ensure an acceptably low probability of ruin, should be deducted from the net cash flow then remaining in the same way as the gearing margin is deducted. The net cash flow then consists of the DAC margin and any further profit margin. That net cash flow is capitalised at the product risk discount rate, i.e. the DAC yields in the examples above, to put a figure on the intangible value of the business.

Average and DAC yields above show that use of market based discount rates may cause error. The average yield is over both new business sales profit and in-force business profit. A risk discount rate derived from aggregate profits is therefore not suitable for valuation purposes for the in-force business.

4 Ancillary Matters

4.1 A Common Valuation Error

For every asset a market value and market yield can be determined. Asset market value equals future asset proceeds discounted at the asset market yield.

Similarly for every liability a market value and market yield can be determined, Future liability outgo discounted at the liability market yield gives the liability market value. This is clearly seen when assets match liabilities, for then the market yields on assets and liabilities are identical.

When assets are not matched to liabilities the yields on assets and liabilities differ. If a present value is calculated for the net cash flow (being the excess of asset proceeds over liability outgo) when the risk discount rates are different for assets and liabilities then the result will be a nonsense.

Suppose $100 is borrowed for 10 years at 8% p.a. and is invested in shares yielding 15% p.a. The expected net cash flow (distributable profit) is $7 p.a. If the net cash flow of $7 p.a. is capitalised at the asset yield of 15% then the apparent value is $35. However the value of assets is $100 and the value of liabilities is $100 so that the net value is nil. The fiction of a value of $35 arises from the error of implicitly discounting both asset proceeds and liability outgo at 15%, which is incorrect for the liability outgo.

This error is prescribed in PS252 paragraph 4.2 "Appraisal values are then determined as the present value of distributable profits at risk adjusted rates of return". Great care must be exercised in discounting cash flows to ensure that the risk discount rate used is appropriate to the components of the cash flow and that any averaging is appropriate. Fictional value may easily be incorporated.

4.2 "Lock-In" Adjustment to Net Worth

It has been suggested in the actuarial literature that to the extent that net worth assets are 'locked in" to meet statutory minimum requirements, such assets should be valued at less than their market value. The discount is to allow for the investment earnings thereon being below the risk discount rate required by the shareholder. The gearing adjusted risk discount rate addresses this issue so that no valuation adjustment is needed.

4.3 Reinsurance

For insurable liabilities the asset maintenance cost may be met by reinsurance. For assets, futures and options may provide similar protection. For self- insurance the asset maintenance margin may be in the premium rate or in the yield on assets. The alternatives must be properly allowed for.

4.4 Superannuation

Suppose that in a defined benefit superannuation scheme the superannuation benefit provided is a multiple of annual salary. If annual salary increases in line with Average Weekly Earnings (AWE) then the risk discount rate needs a 0.5% reduction below the bond rate. If the superannuation fund assets are invested in a share portfolio comparable to the All Ordinaries Index (AOI) then the employer's balance sheet implicitly carries the reserve against adverse fluctuations in the change in AWE and the yield on AOI. At the 97.5% certainty of payment of benefits level the implicit employer fluctuation reserve is 40% of the employees benefit liability if the reserve is held in risk free assets. The extra yield needed on the reserve as compensation for the risk is about 18%.

5 Contrast with Margin on Services

To provide an intuitive basis for the contrast between market values and MoS values consider the example below.

Suppose that, when the mortality rate is 1%, that 100 lives are insured for one year with the benefit of $100 payable only on death and payable at the end of the year of death. Ignoring expenses and mortality fluctuation risk, the expected claim cost of $100 has a value, when the yield on a matching risk free bond is 5%, of $95.24. If a $95.24 face value 5% coupon bond is held then assets and liabilities are financially matched. No asset resilience reserves are required. The market value of the liability is $95.24 and remains at $95.24 even if the assets held are changed to a mismatched position. This is clear since if the liability is transferred to a third party then the third party will require a payment of $95.24, irrespective of the type of assets held. Otherwise arbitrage opportunities exist for either party.

For present illustrative purposes suppose that the premium charged equals $95.24. The above market value equals the MoS policy liability value if and only if the MoS value of future profits equals the market value of the liability less the value of the amount payable discounted at the earnings yield on assets actually held. This holds initially. Since MoS profits may subsequently be reduced or increased this equality need not hold subsequently.

If assets are changed to shares yielding 15% then the initial MoS value is a MoS best estimate of $86.96 plus a MoS value of future profit of $8.28.

Suppose that the mortality rate increases by 9.5% from 1% to 1.095%. The market value of liabilities increases from $95.24 to $104.29.

The MoS value of expected future profits changes by \$86.96 (best estimate liability, basis 1) plus \$8.28 (value of profit, basis 1) less \$86.96 × 1 .095 = \$95.22 (best estimate liability, basis 2) to give a MoS value of \$0.02 for expected future profit on basis 2. The MoS policy liability is then \$95.22+\$0.02=\$95.24. For MoS the cost of increased mortality is offset by the reduction in future profit so that the MoS liability remains unchanged.

The difference between the market value and MoS value reflects the MoS objective of smoothed emergence of profit. The profit reduction emerges year by year in the future. The MoS approach is comparable to use of an amortised book value for assets.

The MoS policy liability value of \$95.24 is less than the market value of liabilities of \$104.29. The MoS value will increase to market value only if the asset portfolio is changed to the asset best matching the liability, i.e. a \$104.28 face value 5% coupon bond.

The significant point is that, whereas on the increase in the mortality rate the market price required to be paid to transfer the liability to a third party increases proportionately, the MoS book value is unchanged.

The market risk discount rate is the PS300 risk free rate of return referable to the expected rate of return on a matched portfolio of investments with minimal risk. This determines a unique market value of liabilities.

6 Conclusions

Suitable discount rates for the calculation of the value of policy liabilities, together with adequate reserve levels for the risks undertaken, may be derived as follows.

(i) Set the liability risk discount equal to:

the yield on the asset portfolio most highly correlated with, i.e. best matching, the policy holder liabilities; less

the margin in respect of residual uncorrelated risks, expressed as a percentage of the liabilities and calculated using collective risk theory, required to give an acceptable probability of ruin.

(ii) Use the distribution of the residual fluctuation in the net of assets less liabilities, when using the asset portfolio best matched to the liabilities, to determine the liability fluctuation reserves allowing liabilities to be met with acceptable probability.

(iii) Further asset liability mismatching fluctuation may be introduced by changing policy reserve assets away from the matched position. Mismatching reserves may be determined allowing liabilities to be met with acceptable probability. The balance of the assets, after providing for the policy liabilities and the reserve for residual fluctuations when assets are matched with liabilities, provides the mismatching reserve. The geared return required on the reserves is then easily determined.

The net asset position of a life office, being the excess of assets at market value over policy liabilities at market value as calculated above, is quite different from the M0S viewpoint of the actuarial profession for which policy liabilities vary in value according to the asset portfolio held.

The effects of systemic and diversifiable risks may be provided for using the general form of the Central Limit Theorems. If the mean of the risk distribution is treated stochastically, so that allowance is made for both undiversifiable systemic risk and diversifiable risk, then the resulting reserve levels exceed those calculated using the common actuarial approach where only diversifiable risk is provided for.

The multi-factor regression models used by investment practitioners may be used to determine the yield on an asset portfolio matching policy liabilities. For insured risks, e.g. death, the lack of correlation with bond markets means that cash or bonds are the matching asset.

Typical gross yields required on asset liability fluctuation reserves are 11% above the matching bond rate for flexible insurance and 16% above the matching bond rate for superannuation products when assets are matched to liabilities.

The geared up return required on asset liability mismatch, i.e. resilience, reserves is typically well above the return yielded by the assets alone.

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Appendix A Theoretical Basis for Risk Discount Rate Determination

A.1 Liability Value and Risk Discount Rate

To provide an intuitive basis for the model developed below consider an example.

Suppose that a unit linked policy has benefits determined in accordance with the movement in the All Ordinaries Accumulation Index. The policy liability has a risk discount rate and market value which exists independently of the asset portfolio held. The risk discount rate and market value of policy liability are those of the corresponding matching asset portfolio.

When assets match liabilities then assets and liabilities have a correlation $r = 1$. Denote the value of the matching asset portfolio by $\boldsymbol{A'}_r$.

If the life office mismatches assets and liabilities by confining investment to a small number of stocks then there is a residual tracking error, denoted by $\boldsymbol{A'}_{(1-r)}$. $\boldsymbol{A'}_{(1-r)}$ is not correlated with $\boldsymbol{A'}_r$ and the expected value is $\mathrm{E}[\boldsymbol{A'}_{(1-r)}] = 0$. For a mismatched asset portfolio of value $\boldsymbol{A}$, $\boldsymbol{A} = \boldsymbol{A'}_r + \boldsymbol{A'}_{(1-r)}$. Since $\mathrm{E}[\boldsymbol{A'}_{(1-r)}] = 0$, the yield on $\boldsymbol{A}$ equals the yield on $\boldsymbol{A'}_r$, i.e. the yield on the portfolio most correlated with the liabilities.

For a life office let the market value of assets be $\boldsymbol{A}$ and the market value of liabilities be $\boldsymbol{L}$, where both $\boldsymbol{A}$ and $\boldsymbol{L}$ are stochastic variables. Denote the means of $\boldsymbol{A}$ and $\boldsymbol{L}$ by $\mathrm{E}[\boldsymbol{A}]$ and $\mathrm{E}[\boldsymbol{L}]$ and let $\mathrm{E}[\boldsymbol{A}] = \mathrm{E}[\boldsymbol{L}]$. Denote the variance of $\boldsymbol{A} - \boldsymbol{L}$ by $\mathrm{V}[\boldsymbol{A} - \boldsymbol{L}] = 2(1 - r_{\boldsymbol{A},\boldsymbol{L}})$ where $r_{\boldsymbol{A},\boldsymbol{L}}$ is the correlation coefficient of $\boldsymbol{A}$ and $\boldsymbol{L}$.

When $\boldsymbol{A}$ and $\boldsymbol{L}$ are deterministically matched then $\mathrm{V}[\boldsymbol{A} - \boldsymbol{L}] = 0$. When $\boldsymbol{A}$ and $\boldsymbol{L}$ are stochastically matched then $\mathrm{V}[\boldsymbol{A} - \boldsymbol{L}]$ is a minimum. Correspondingly $r_{\boldsymbol{A},\boldsymbol{L}} = 1$ or $r_{\boldsymbol{A},\boldsymbol{L}}$ is a maximum.

For given $\boldsymbol{L}$ denote by $\boldsymbol{A'}$ the asset portfolio minimising $\mathrm{V}[\boldsymbol{A} - \boldsymbol{L}]$ and denote by r the corresponding maximum correlation. Orthogonally decompose $\boldsymbol{A'}$ into two parts $\boldsymbol{A'}_r$ and $\boldsymbol{A'}_{(1-r)}$ which are correlated with $\boldsymbol{L}$ and uncorrelated with $\boldsymbol{L}$ respectively. $\boldsymbol{A'}_r$ and $\boldsymbol{A'}_{(1-r)}$ are then by definition uncorrelated; as well $\mathrm{E}[\boldsymbol{A'}_{(1-r)}] = 0$. Similarly, define $\boldsymbol{L'}_r$

and $\boldsymbol{L'}_{(1-r)}$ which are also by definition uncorrelated. Moreover, since by definition E[$\boldsymbol{A}$] = E[$\boldsymbol{L}$] and $\boldsymbol{A'}_r$ and $\boldsymbol{L'}_r$ have correlation 1, $\boldsymbol{A'}_r = \boldsymbol{L'}_r$

If assets and liabilities are deterministically matched and if E[$\boldsymbol{A}$] = E[$\boldsymbol{L}$] then $\boldsymbol{A} = \boldsymbol{L}$ and the risk discount rate giving the capitalised value $\boldsymbol{L}$ is the internal rate of return on asset proceeds giving the market value $\boldsymbol{A}$. Note that for deterministic matching $\boldsymbol{A} = \boldsymbol{A}_r$ and $\boldsymbol{L} = \boldsymbol{L}_r$.

Similarly, if assets and liabilities are stochastically matched and if E[$\boldsymbol{A}$] = E[$\boldsymbol{L}$] then E[$\boldsymbol{A'}$] = E[$\boldsymbol{L}$] and the risk discount rate giving the capitalised value E[$\boldsymbol{L}$] is the internal rate of return on asset proceeds giving the market value E[$\boldsymbol{A'}$] . This gives a unique risk discount rate for a given $\boldsymbol{L}$ so that $\boldsymbol{L}$ has a unique market value, independent of $\boldsymbol{A}$ as distinct from $\boldsymbol{A'}$.

A.2 Capital Requirement: Liability Risk

Suppose that assets and liabilities are mismatched in the unit linked policy example above. Fluctuation reserves C_E must be held to ensure that liabilities can be met, with probability $R_E = 99.9\%$ for example. Since the residual tracking error equals $\boldsymbol{L} - \boldsymbol{A}'$, reserves C_E satisfy $\Pr(\boldsymbol{L} - \boldsymbol{A}' < C_E) = 99.9\%$

In general, for existing business with expected value of liabilities $L = \mathrm{E}[\boldsymbol{L}]$ so that $\mathrm{E}[\boldsymbol{A}] = L$, capital denoted C_E is required as fluctuation reserves to cover the fluctuation risk inherent in $\boldsymbol{L} - \boldsymbol{A}'$. Denote R_E the probability (e.g. 99.9%) that reserves C_E are adequate. Formally $\Pr(\boldsymbol{L} - \boldsymbol{A}' < C_E) = R_E$.

To use this result note that

$$
\begin{aligned}
&\Pr(\boldsymbol{L} - \boldsymbol{A}' < C_E)\\
&= \Pr((\boldsymbol{L} - \boldsymbol{L}_r) - (\boldsymbol{A}' - \boldsymbol{A}_r) < C_E)\\
&= \Pr(\boldsymbol{L}_{(1-r)} - \boldsymbol{A}'_{(1-r)}) < C_E)\\
&=1 - \Pr(\boldsymbol{L}_{(1-r)} - \boldsymbol{A}'_{(1-r)}) > C_E)\\
&=1 - \Pr((\boldsymbol{L}_{(1-r)} - L) + (A - \boldsymbol{A}'_{(1-r)})) > C_E)\\
&\approx 1 - [\Pr(\boldsymbol{L}_{(1-r)} - L) > C_E) + \Pr(A - \boldsymbol{A}'_{(1-r)}) > C_E)]\\
&\text{when } C_E \text{ is small}
\end{aligned}
$$

i.e. for practical purposes reserves are lost if either:

actual liabilities exceed expected liabilities by more than the reserves, or

actual assets are less than expected assets by more than the reserves,

so that C_E is approximately the greater of the values of C_E calculated from the two equalities $\Pr(\boldsymbol{L}_{(1-r)} - L) > C_E = R_E$ and $\Pr(A - \boldsymbol{A}'_{(1-r)}) > C_E) = R_E$.

The above determines the minimum reserves which may be held for existing business. On a market efficiency criterion this may be taken as the actual level and any excess allocated to general surplus reserves. Hence the capital requirement in respect of the liability risk is known.

A.3 Capital Requirement: Asset Liability Mismatch Risk

If the actual asset portfolio $\boldsymbol{A}$ held differs from $\boldsymbol{A'}$ then the mismatch $\boldsymbol{A'} - \boldsymbol{A}$ introduces a further source of fluctuation risk against which fluctuation reserves C_A must be held, adequate with probability R_E. Formally, C_A satisfies $\Pr(\boldsymbol{A'} - \boldsymbol{A} < C_A) = R_E$.

The provider of the capital C_A requires a return on the mismatch reserve C_A calculated according to the gearing formula:

k'_e	is the market yield on an ungeared asset
k_d	is the market yield on debt secured on the asset
B	is the market value of debt secured on the asset
S	is the market value of the residual equity
k_e	is the market yield on the geared asset

then

$$k_e = k'_e + (B / S)\,(k'_e - k_d)$$

where in the present case $B = L$ and $S = C_A$.

The above means that assets C_A should be included in the balance sheet at market value without any adjustment. Market value exists independently of the business of the life office. Assets include "net worth" assets i.e. the assets supporting the excess of asset value over liabilities value.

A.4 "Law of Averages" Effects

From A.l, A.2 and A.3 above the liability risk discount rate and the capital requirement C_E in respect of the liabilities does not depend on the actual asset portfolio $\boldsymbol{A}$. Rather C_E depends on $\boldsymbol{A}'$ and $\boldsymbol{L}$.

For a large number of risks diversification reduces the fluctuation in both $\boldsymbol{A}'$ and $\boldsymbol{L}$. If total variance is divided into diversifiable and systemic components then increasing the number of assets, or increasing the number of individual risks in the liability, reduces the diversifiable component of the variance until in the limit only the systemic variance component remains.

The diversifiable and systemic sources of variance may be characterised as

(i) diversifiable fluctuation i.e. fluctuation in asset or liability experience for a particular portfolio about the applicable distribution mean given the mean;

(ii) systemic fluctuation i.e. fluctuations in the mean of the distribution function underlying the experience.

This can be seen in the general form of the Central Limit Theorems. For a single risk let $F(x|\mu)$ be the distribution of the probability of the risk for a given mean μ and let $G(\mu)$ be the distribution of μ the mean probability of the risk. Assume sufficient regularity in F and G for the existence of their densities $f(x|\mu)$ and $g(\mu)$. Let $\boldsymbol{X}$ be a random variable representing the risk, then $\boldsymbol{X}$ has the distribution $H(x) = \int_\mu f(x|\mu)\cdot g(\mu)d\mu$. Let $\boldsymbol{S_n}$ be the sum over n random variables $\boldsymbol{X}$. Then the distribution of $\boldsymbol{S_n} / n^{1/2}$ tends to $\int_\mu n(x|\mu)\cdot g(\mu)d\mu$, where $n(x|\mu)$ is a "normal" distribution with mean μ. Note that when μ is fixed $g(\mu)$ is a point density and the distribution of $\boldsymbol{S_n} / n^{1/2}$ then tends to $n(x|\mu)$, the elementary form of the Central Limit Theorem.

The distribution $\int_\mu f(x|\mu)\cdot g(\mu)\mathrm{d}\mu$ is that of the systemic risk. The diversifiable component of risk is uncorrelated with the systemic risk. Note particularly that this systemic risk is not normally distributed unless $g(\mu)$ is a point density. The common assumption that the systemic risk is normally distributed is not appropriate and is the source of much error.

From arbitrage theory there is no economic return for bearing diversifiable risk, so for pricing fluctuation risk that risk is taken as being wholly systemic.

A.5 Asset Maintenance Costs

Risk discount rates allow only for financial risk. Moreover, the return receivable for financial risk is only in respect of undiversifiable financial risk. The cost of other types of risk should be added to the liability cash flow as an asset maintenance cost. This is because other types of risk are characterised by zero correlation with financial markets, so that there is no undiversifiable financial risk incurred to earn an extra return above the appropriate financial risk adjusted rate. How may asset maintenance costs be determined?

Since De Moivre’s time in the 18th century it has been known that an insurance company must charge a contingency margin, in addition to the average cost of the risk, if ruin is to be avoided. If a profit or contingency margin is not provided for then random fluctuations will eventually result in the capital of the life office being lost. The asset maintenance cost provides that margin.

For example, mortality experience has zero correlation with economic variables so the appropriate discount rate for financial risk is the bond rate for the average policy term. For the whole Australian population diversifiable mortality risk is absent; calculation of the variation around the mortality trend line (technically equivalent to treating $g(\mu)$ as a point density) gives mortality as normally distributed with a standard deviation of 0.00047 and current trend mean mortality rate of 0.0071. If initial reserves are 20% of expected claims (adequate against one year’s fluctuation with 99.9% probability) and the acceptable probability of ruin is 2.5% then a contingency margin of 4.3% is required. This contingency margin is the cost of the guarantee that death benefits can be paid; thus it is an asset maintenance cost in respect of a non financial risk. For computational simplicity the asset maintenance cost of 4.3% may be deducted from the bond rate for the purpose of discounting expected future claims experience. However that resultant rate used for discounting is conceptually a combination of two distinct parts.

The problem is well known as the ruin problem of collective risk theory. The density $g(\mu)$ is the density over time of the mean of the relevant population subject to the risk. The general solution is merely a particular instance of the application of statistical queuing theory.

A.6 Methodology

The result is that the method for determining the policy liability, reserve levels required, and appropriate risk discount is as follows.

- determine the risk discount rate appropriate to the liabilities as the yield on the asset portfolio $\boldsymbol{A'}$ most highly correlated with the policy holder liabilities $\boldsymbol{L}$;
- in respect of the residual risks in $\boldsymbol{A'} - \boldsymbol{L}$ use collective risk theory to calculate the asset maintenance margin required to give an acceptable probability of ruin;
- calculate the value of the liabilities as the value at the yield on $\boldsymbol{A'}$ of the expected payments to policy holders $\boldsymbol{L}$ plus the asset maintenance cost M.
- using the distribution of $\boldsymbol{A'} - \boldsymbol{L} - M$ calculate the fluctuation reserve R_E required to allow liabilities to be met with acceptable probability;
- for an actual asset portfolio $\boldsymbol{A}$ calculate the reserve R_A required to allow fluctuations in $\boldsymbol{A'} - \boldsymbol{A}$ (where $\boldsymbol{A'} = \boldsymbol{L} + M$) to be met with acceptable probability, as well R_A is then credited with the appropriate geared up return;
- total reserves then comprise:

  $\boldsymbol{L} + M$ for policy liabilities, plus

  $R_E + R_A$ for fluctuation reserves in respect of $\boldsymbol{A'} - \boldsymbol{L} - M$ and $\boldsymbol{A'} - \boldsymbol{A}$;
- total assets required are then $\boldsymbol{L} + M + R_E + R_A$ and net assets are $R_E + R_A$.

Net life office assets equals the excess of assets at market value over policy liabilities at market value as determined above. The Australian actuarial practice of valuing liabilities at a risk discount rate varying with the yield on assets gives a different result.

Appendix B Risk Adjusted Return Required on Sales

To quantify the financial risk inherent in a sales profit centre consider for example sales in Australia of investment account business over the last 10 years for the whole life insurance industry as given in Table 9a in the Life Insurance Commissioners Quarterly Statistical Bulletins over the period 1984 to 1994. Sales volumes over the last 10 years have moved counter cyclically to stock market movements. When share prices fell investment account sales rose and when share prices rose investment account business fell. The standardised regression coefficient of sales volumes against the yield on the All Ordinaries Index (see Australian Stock Exchange Monthly Index Analysis) was -0,85 so that using a simple CAPM model and a 6.5% risk premium a yield about 5% less than the bond rate is appropriate for financial risk. Because the rate applies only for the short period between the times of marketing expenditure and sale such a simple approximation is acceptable.

What contingency margin should be provided against the risk of ruin of the sales profit centre? Sales profit, i.e. the excess of the value of DAC generated over sales and marketing costs, will fluctuate from year to year. Suppose that an amount of initial capital is allocated to the sales profit centre. Queuing theory can be used to quantify the ruin probability. For the whole investment account business of the Australian life insurance industry over the last 10 years (ibid.), the distribution of sales can be modelled by a gamma distribution with mean \$770 million, standard deviation \$290 million. Using this distribution in the ruin probability formula (see Prabhu page 100, as used in Appendix AS) with a ruin probability of 2.5%, for reserves of one year's DAC sold (i.e. 2.8 standard deviations) then the required margin is 20% of DAC generated. This is a commercially acceptable position.

Results for a sales profit centre for life office investment account business are:

- a yield 5% below the bond rate is acceptable for financial risk, and
- a margin of 20% of expected DAC generated (equivalent to a 25% margin on sales cost) is required.

Appendix C Risk Adjustment to Return for Surrender Risk

To quantify life office systemic surrender risk consider the experience of the aggregate of all life offices. Over the last ten years the investment account business of the whole life insurance industry has shown an average annual surrender rate of 15%. (See Tables 3A and 9a in the Life Insurance Commissioners Quarterly Statistical Bulletins over the period 1984 to 1994.) For life insurance business with a wide spread of ages deaths will have a termination rate about 0.5%. Hence, regarding the death rate as relatively insignificant, no adjustment to the benchmark life insurance 15% surrender rate to allow for deaths included in that rate has been made.

Surrender rates as a percentage of in force investment account business may be derived from Tables 3A and 9a in the Life Insurance Commissioners Quarterly Statistical Bulletins over the period 1984 to 1994. The surrender rate has a quarterly mean of 3.5% and standard deviation of 5.3%. For the same period the All Ordinaries Accumulation Index gave a mean quarterly yield of 4.3% and standard deviation of 10.7%. The standardised regression coefficient (beta) of the surrender rate on the All Ordinaries yield is 0.05. (In this context use of multiple factors gives spurious accuracy.) Grossed up for imputation credits the regression gradient is 11%. Hence the yield adjustment is $0.05 \times 11\% = 0.5\%$ gross or $0.05\% \times 0.64 = 0.3\%$ net of tax at 36%.

By way of comparison the mean bank bill rate over the 10 years was 2,9% per quarter (12.1% per annum), whilst the all bonds index had a mean yield of 3.2% (13.4% per annum) with standard deviation 3.3%. (See Commonwealth Bank Bond Indices and Yields.) The standardised regression coefficient of the all bonds index yield on the All Ordinaries Index yield was $0.085 \times 11 = 13.0\%$ as compared with the 13.4% observed.

An increase in the risk discount rate of 0.5% to 1.0% to allow for surrender risk is consistent with the results derived by Mehta (1992) for UK experience.

Appendix D Risk Adjustment to Return for Mortality Risk

Based on mortality statistics for the whole Australian population (see ABS Social Indicators Table 1.5.1), i.e. the population mortality rates for the period 1901 to 1994 so that no diversifiable risk is included, the standard deviation of the population mortality rate relative to the population mortality rate trend line is 6.5% of the current trend line rate. At the 99.9% confidence level, i.e. at 3 standard deviations, this gives a required mortality fluctuation reserve of 20% of expected claims against one year’s possible fluctuation. Hence risk-free asset reserves of 20% of expected claims are required to cover one years expected fluctuation in mortality experience.

If mortality fluctuation reserves against undiversifiable risk are held at the 3 standard deviation level it is informative to consider the probability of ruin inherent in a given profit margin. Suppose that a 2.5% probability of ruin is acceptable. The 2.5% level is consistent with the exit rate of life offices from the industry in recent years. Ruin theory formulae, using the distribution around the trend line of Australian population mortality, give a required profit margin of 4.3% of expected claims. A further contingency margin is required against diversifiable risk, Hence to ensure preservation of the mortality fluctuation reserves a margin of at least 20% of that reserve is required i.e. at least a 4% additional return on reserves.

If reserves against undiversifiable risk are held at the 2 standard deviation level then the required profit margin rises to about 6% of expected claims using the same 2.5% probability of ruin.

For investment account business the sum at risk is the value of the DAC asset. If this averages 5% of policy holder balances then the cost. is about 0.2% to 0.3%. A reduction in the discount rate for liabilities of about 0.2% suffices to provide for this risk.

Appendix E Capital guarantee: Margin Against Ruin

For non-participating guaranteed rate investment account business, suppose that interest is earned at rate ***I*** (a stochastic variable being the Commonwealth All Bonds (non-rebateable) Index), ***U*** is the initial reserve level, and interest is credited at rate ***i*** on non-participating policies. The Commonwealth All Bonds (non-rebateable) Index quarterly yields from 1979 to 1994 can be modelled with a gamma distribution $\Gamma(x + 5\%, c = 0.85, r = 6)$. If initial capital is \$100 and reserves are \$50, then the probability that capital and reserves will fall below \$100 is 2.5%. For a mean return of 2% per quarter, if 1% per quarter is credited on non-participating business capital of \$100, then the required reserve increases to \$55 for an unchanged ruin probability. It is no surprise that little business is written on such terms.

The above analysis can be extended to participating investment account business. Let ***I*** be the earning rate and ***C*** be the crediting rate, where both ***I*** and ***C*** are stochastic variables. If ***I*** is the Commonwealth All Bonds (non-rebateable) Index quarterly yields, and ***C*** is the Bank Bill Index quarterly yields (corresponding to the asset matching capital guaranteed investment account business), then over the last decade ***I* – *C*** can be modelled with a normal distribution with mean 0.07% and standard deviation of 2.0%. The mean of ***I* – *C*** provides the contingency margin available to protect against ruin. The margin may fluctuate over time since the mean is not stable.

For the Commonwealth All Bonds (non-rebateable) Index, Circular 273 reserves were 8.5% of assets. If such a reserve level was held then to satisfy a 2.5% probability of ruin criterion the margin ***I* – *C*** needed to be 1.0% or more.

Appendix F Pricing Models

F.1 Capital Asset Pricing Model

The Capital Asset Pricing Model (CAPM) may be used to estimate discount rates. In an Australian context CAPM has several practical difficulties.

(i) The small number of stocks in the All Ordinaries Index and concentration of value in a few large stocks, e.g. BHP, means estimates of CAPM Betas are often biased. Technically difficult techniques are needed to address that bias so that biased estimates are commonly used.

(ii) Uncertainty of benchmark gives results which in practice may contradict those expected from theory. E.g. using data 1990 to 1995, the All Ordinaries Index may be divided into All Industrials (Beta = 0.92) and All Resources (Beta = 1.16) with Betas relative to the All Ordinaries Index. CAPM indicates All Industrials should have a lower yield than All Resources. In practice the opposite is observed so that CAPM based yield estimates are subject to some doubt. However, noting that as a broad generalisation All Resources stocks are traded internationally whereas All Industrials stocks are not, relative to the MSCI World Index, All Industrials (Beta = 0.56) and All Resources (Beta = 0.31) give results consistent with CAPM theory.

(iii) Significant factors are omitted in a single factor model. Size (stock market capitalisation) is a statistically much more significant indicator of yield than volatility as popularised by CAPM. E.g. regional banks, formerly building societies, are characterised by narrow product ranges and geographical concentration which decouple their performance from the Australian economy and result in low Beta factors. Major banks with a broad product range and geographical diversification have performance coupled to the Australian economy and hence have higher Beta factors. However, contrary to what is expected using CAPM, regional banks give higher yields than major banks. Liquidity, or its proxy measure market capitalisation, appears to be the additional factor explaining the yield relativities.

F.2 Multi-Factor Models

Multi-factor models, e.g. BARRA, ANZ, etc., are routinely used by investment professionals to estimate yields for investment purposes. These models allow portfolio tracking errors to be kept acceptably small. Such models if used to estimate yields for the purpose of capitalising future policy holder liabilities as at a balance sheet date give the same answer as that for the yield on an asset/liability portfolio having minimum variance. This is a consequence of constructing the asset portfolio to have the same regression coefficients on the statistically significant independent factors as does the liability portfolio. Note again that asset maintenance costs are in this context added to the liability cash flow: they are not allowed for in the yield.

Such multi-factor models are usable for the purpose of determining the financial risk discount rate appropriate to liabilities.

Appendix G Individual Examples

G.1 Funeral Bonds

Suppose that a marketing expenditure of \$4 in a sales profit centre produces a sale of \$100. If the DAC profit margin is 25% of marketing expenditure then DAC of \$5 is created. If this is sold to the parent life office four months after the marketing expenditure then the payment required is \$5.05 allowing for interest at 8% – 5% = 3%.

The DAC of \$5.05 is recovered from future cash flows. Cash flows are increased each year by 6% bonus and decreased by 10% due to mortality, arbitrarily limited to 30 years for present purposes. The resulting annuity value is 5.98 at 12% interest. Hence the annual recoup is initially \$5.05/5.98 = \$0.84 per \$100 sale.

The annual recoups are then \$0.84, \$0.80, \$0.76, ... allowing for a 6% bonus rate and a 10% mortality rate.

The internal rate of return on a \$4.00 marketing outlay and annual recoups of \$0.84, \$0.80, \$0.76, ... is 16.4%.

G.2 Flexible Insurance

Suppose that a marketing expenditure of \$4 in a sales profit centre produces a sale of \$100. If the DAC profit margin is 25% of marketing expenditure then DAC of \$5 is created. If this is sold to the parent life office four months after the marketing expenditure then the payment required is \$5.05 allowing for interest at 8% – 5% = 3%.

The DAC of \$5.05 is recovered from future cash flows. Cash flows are increased each year by 4% bonus and decreased by 15% due to surrenders. The resulting annuity value is 4.14 at 12.5% interest. Hence the annual recoup is initially 55.05/4.14 = \$1.22 per \$100 sale.

The annual recoups are then \$1.22, \$1.08, \$0.95, ... allowing for a 4% bonus rate and a 15% surrender rate.

The internal rate of return on a \$4.00 marketing outlay and annual recoups of \$1.22, \$1.08, \$0.95, ... is 18.9%.

G.3 Superannuation/Deferred Annuity Bonds

Suppose that a marketing expenditure of \$3.20 in a sales profit centre produces a sale of \$100. If the DAC profit margin is 25% of marketing expenditure then DAC of \$4.00 is created. If this is sold to the parent life office four months after the marketing expenditure then the payment required is \$4.04 allowing for interest at 8% – 5% = 3%.

The DAC of \$4.04 is recovered from future cash flows. Cash flows are increased each year by 5% bonus and decreased by 33% due to surrenders. The resulting annuity value is 2.37 at 12.5% interest. Hence the annual recoup is initially \$4.04/2.37 = \$1.70 per \$100 sale.

The annual recoups are then \$1.70, \$1.20, \$0.84, ... allowing for a 5% bonus rate and a 33% surrender rate.

The internal rate of return on a \$3.20 marketing outlay and annual recoups of \$1.70, \$1.20, \$0.84, ... is 23.5%.

G.4 Solvency Reserves

Using the formula in Appendix A.3, with $k'_e = 8\%$ and $k_d = 8.2\%$, together with policy holder balance $B = 100$ and solvency reserves $S = 1$, then

$$\begin{aligned} k_e &= k'e + (B/S)\,(k'_e - k_d) \\ &= 8\% + (100/11)\,(8.2\% - 8\%) \\ &= 8\% + 20\% \\ &= 28\% \end{aligned}$$